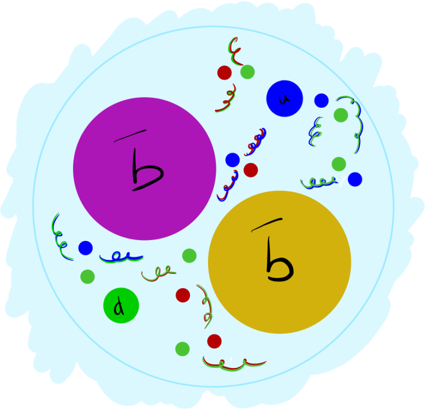
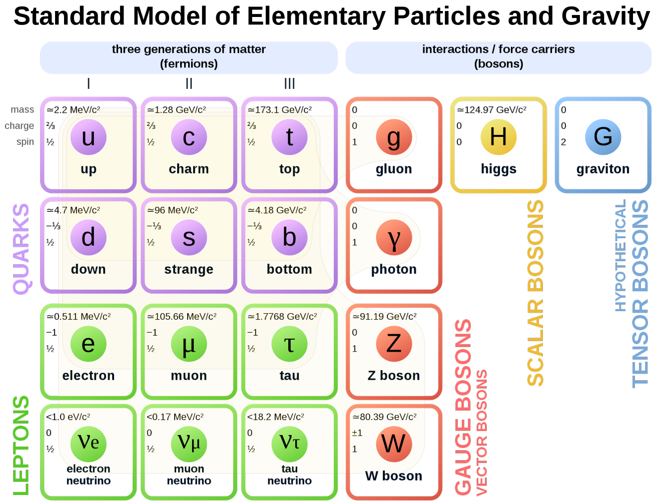
EXOTIC STATES ON A LATTICE

Brian Colquhoun

February 23 2021



The Standard Model



Standard Model of Elementary Particles and Gravity

	three generations of matter (fermions)			interactions / force carriers (bosons)		
	I	II	III			
mass charge spin	≃2.2 MeV/c^2 ⅔ ½ u **up**	≃1.28 GeV/c^2 ⅔ ½ c **charm**	≃173.1 GeV/c^2 ⅔ ½ t **top**	0 0 1 g **gluon**	≃124.97 GeV/c^2 0 0 H **higgs**	0 0 2 G **graviton**
QUARKS	≃4.7 MeV/c^2 −⅓ ½ d **down**	≃96 MeV/c^2 −⅓ ½ s **strange**	≃4.18 GeV/c^2 −⅓ ½ b **bottom**	0 0 1 γ **photon**	SCALAR BOSONS	HYPOTHETICAL TENSOR BOSONS
	≃0.511 MeV/c^2 −1 ½ e **electron**	≃105.66 MeV/c^2 −1 ½ μ **muon**	≃1.7768 GeV/c^2 −1 ½ τ **tau**	≃91.19 GeV/c^2 0 1 Z **Z boson**	GAUGE BOSONS VECTOR BOSONS	
LEPTONS	<1.0 eV/c^2 0 ½ νe **electron neutrino**	<0.17 MeV/c^2 0 ½ νμ **muon neutrino**	<18.2 MeV/c^2 0 ½ ντ **tau neutrino**	≃80.39 GeV/c^2 ±1 1 W **W boson**		

The Standard Model & QCD

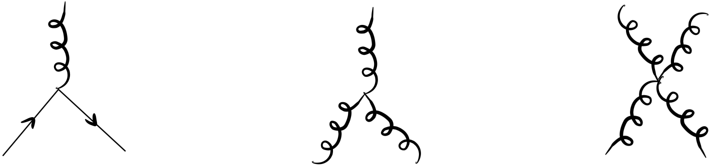
QCD Lagrangian

$$L = \sum_f \bar{\psi}_{f,i} \left(i\gamma_\mu D^\mu - m_f \delta_{ij} \right) \Psi_{f,j} - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

with $D^\mu_{ij} = \partial_\mu \delta_{ij} + i g_s t^a_{ij} A^\mu_a$

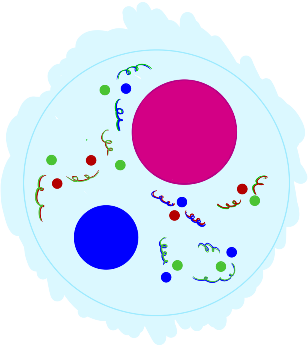
and $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_s f_{abc} A^b_\mu A^c_\nu$

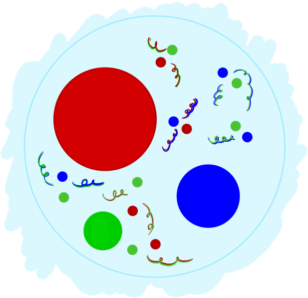
- Quarks are massive particles
- Gluons are massless particles
  - Gauge bosons in QCD self-interact, unlike photons in QED



QCD

Colour confinement in QCD means quarks form colourless bound states, hadrons.





- ★ Mesons: $q\bar{q}$
- ★ Baryons: qqq or $\bar{q}\bar{q}\bar{q}$

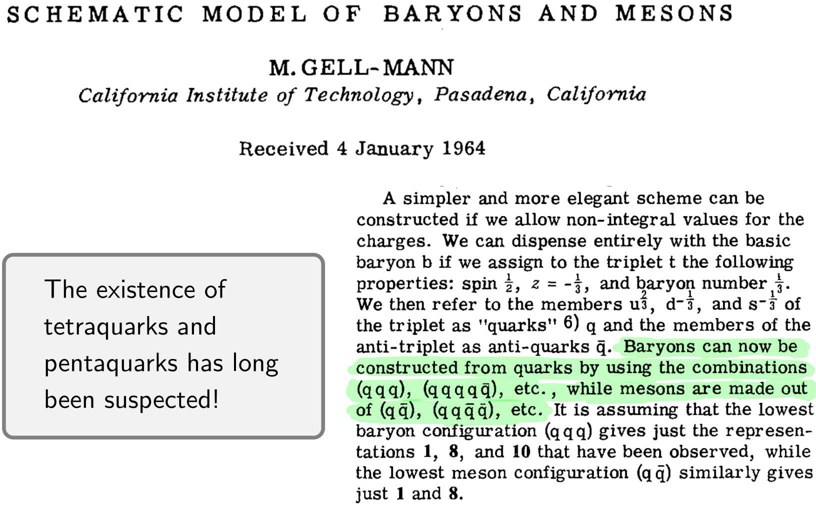
Exotic states on a lattice

Outline of Talk

1. Tetraquarks in the Standard Model
2. Overview of Lattice Field Theory
3. Tetraquarks on the Lattice
4. Other Exotics on the Lattice

Tetraquarks in the Standard Model

Tetraquarks



SCHEMATIC MODEL OF BARYONS AND MESONS

M. GELL-MANN
California Institute of Technology, Pasadena, California

Received 4 January 1964

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" 6) q and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations $(q\,q\,q)$, $(q\,q\,q\,q\,\bar{q})$, etc., while mesons are made out of $(q\,\bar{q})$, $(q\,q\,\bar{q}\,\bar{q})$, etc. It is assuming that the lowest baryon configuration $(q\,q\,q)$ gives just the representations **1**, **8**, and **10** that have been observed, while the lowest meson configuration $(q\,\bar{q})$ similarly gives just **1** and **8**.

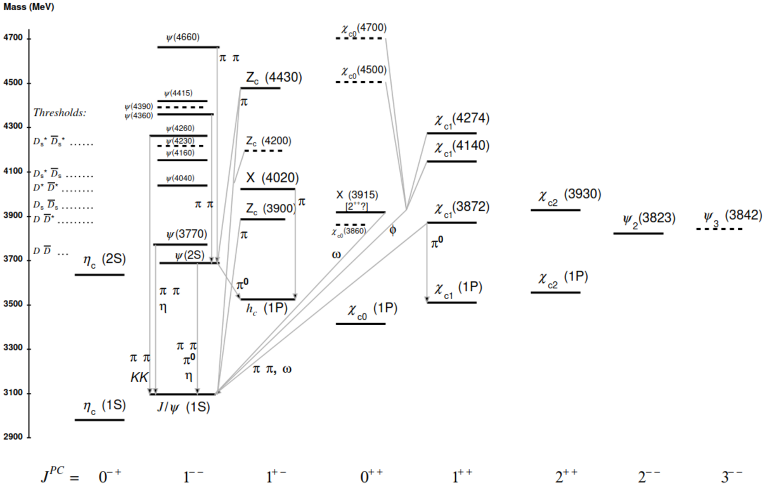
The existence of tetraquarks and pentaquarks has long been suspected!

Some hints of tetraquarks?

In the last two decades there are have been a whole host of mesons with heavy quark content that do not seem to fit the quark model picture. They have been referred to as "XYZ" states.

- Belle Collaboration: χ_{c1} [formerly $(X(3872))$] in 2003 [hep-ex/0309032].
  - Also seen by:
    - BaBar
    - CDF
    - D0
    - LHCb
- Belle Collaboration: $Z_c(4430)$ [0708.1790].
  - This time, BaBar did not find it.
  - LHCb found it in 2014 [1404.1903]
- But typically, many "XYZ" states only seen by single experiment

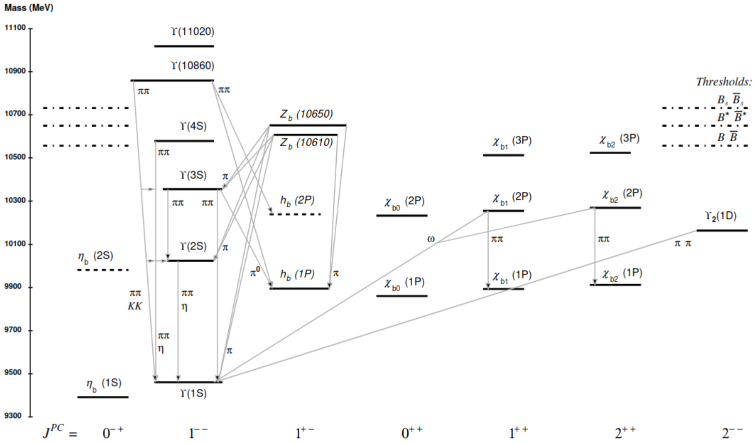
Charmonium spectrum



2020 Review of Particle Physics. P.A. Zyla et al. (Particle Data Group)

Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

Bottomonium spectrum



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Reproduction difficulty

The D0 Collaboration reported evidence for a $B_s\pi^\pm$ state they call $X(5568)$. They suggest:

- 4 quark state, $us\overline{db}$
- Possible $J^P = 0^+$
- Alternatively, $B_s^*\pi^\pm$, $B_s \to B_s^0\gamma$ $\Rightarrow J^P = 1^+$

D0 Collaboration [1607.05214],[1712.10176]

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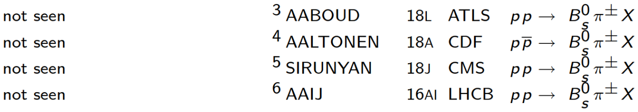
However:



not seen	[3] AABOUD	18L	ATLS	$pp \to B_s^0\pi^\pm X$
not seen	[4] AALTONEN	18A	CDF	$p\overline{p} \to B_s^0\pi^\pm X$
not seen	[5] SIRUNYAN	18J	CMS	$pp \to B_s^0\pi^\pm X$
not seen	[6] AAIJ	16AI	LHCB	$pp \to B_s^0\pi^\pm X$

$cc\bar{c}\bar{c}$?

LHCb discovers a new type of tetraquark at CERN

The LHCb collaboration has observed an exotic particle made up of four charm quarks for the first time

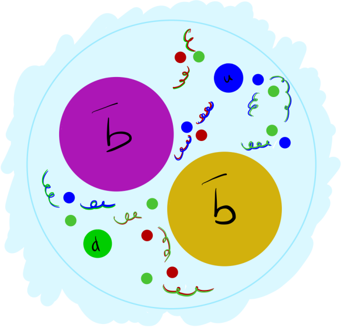
1 JULY, 2020

Illustration of a tetraquark composed of two charm quarks and two charm antiquarks, detected for the first time by the LHCb collaboration at CERN (Image: CERN)

- ★ Last year the LHCb Collaboration reported on what could be a tetraquark [2006.16957]
- ★ The authors of the paper point out that interpretations *other* than a tetraquark are not ruled out
  - ▶ author of the headline was a bit more certain

Diquarks

- ★ Idea: diquarks, qq or $\bar{q}\bar{q}$ pairs
- ★ Not colourless, so not physical.
- ★ But combining two colours is equivalent to the anti-colour of the remaining colour, e.g., $r + b = \bar{g}$



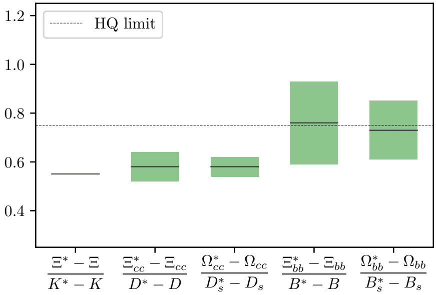
- ★ We are interested in:
  - ▶ light diquarks in a colour $\bar{3}_c$, flavour $\bar{3}_f$ and spin 0 configuration
    - ○ "good light diquark"
  - ▶ heavy diquarks in a colour 3_c configuration

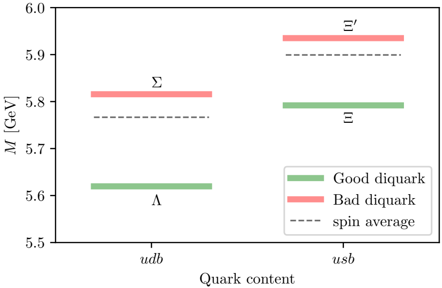
The term "good diquark" is of Jaffe's invention, for a nice review: [hep-ph/0409065]

Information from baryons and mesons

- ★ Ordinary baryon and meson spectra can provide constraints for models
- ★ $\bar{Q}\bar{Q}$ serves as nearly static colour source, like a single Q in a baryon

Numbers from PDG & [1409.0497]

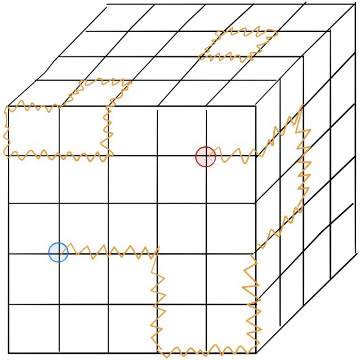




- ★ Baryon spectrum suggests “good” light diquarks result in strong attraction.
- ★ Lighter quark mass → stronger attraction

Overview of Lattice Field Theory

Lattice QCD

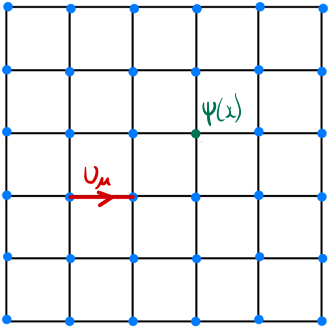


- ★ Lattice QCD discretises spacetime onto 4D lattice.
- ★ Applied to many areas of particle physics, e.g.:
  - ▶ Meson & baryon spectra
  - ▶ Decay constants
  - ▶ Quark masses
  - ▶ CKM matrix elements
  - ▶ α_s
- ★ Can be applied to exotic matter: tetraquarks, exotic mesons, dark matter.
  - ▶ Can be more difficult

Lattice QCD

On a lattice:

- ★ Quarks, $\Psi(x)$, live on lattice sites
- ★ Gluons $U_\mu \in SU(3)$ connect sites
- ★ Typically $L^3 \times T$; $T \sim 2 \times L$
- ★ Lattice period or antiperiodic in L, T



Some other modifications:

- ★ integrals → sums
- ★ derivatives → finite differences

Measurement

In lattice QCD we want to get expectation values from the path integral:

$$\langle\mathcal{O}\rangle = \frac{1}{Z}\int\mathcal{D}[U_\mu]\mathcal{D}[\psi,\bar{\psi}]\mathcal{O}[U_\mu,\psi,\bar{\psi}]e^{-S_E[U_\mu,\psi,\bar{\psi}]}$$

with $S_E = S_g + S_F$

We can split this into parts:

- Generate gauge fields with Monte Carlo methods
- Generate quark propagators
- Contract propagators

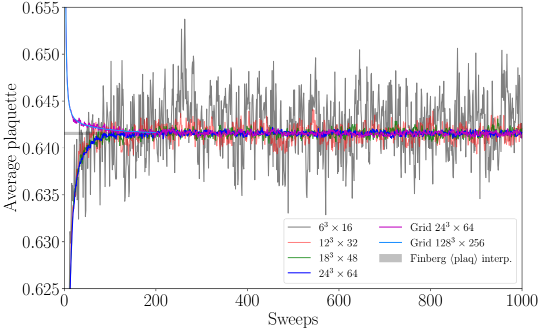
Each stage is computationally expensive!

Compute Canada's
Niagara cluster

Generating fields

To generate gauge fields, Monte Carlo methods to update $U_\mu(x)$. In practice:

- ★ update until lattice thermalized (e.g. check plaquette)
- ★ save every N configs where N large enough to avoid autocorrelations
- ★ Typical ensemble: $\mathcal{O}(100)$, $\mathcal{O}(1000)$ configurations



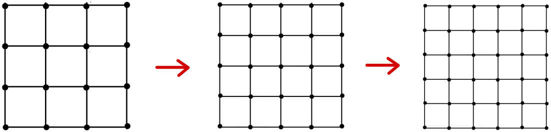
Measurement of average:

$$P_{\mu\nu}(x) = U_\mu(x)U_\nu(x+a\hat{\mu})U^\dagger_\mu(x+a\hat{\nu})U^\dagger_\nu(x)$$

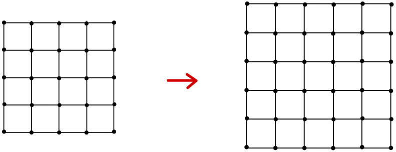
Additional choice: quenched approximation, i.e., exclude effects of sea quarks. This is now relatively rare, most lattice calcs $n_f = 2+1$, $2+1+1$, but more expensive. Requires $\det(M)$, $M = i\not{D} - m$

Lattice sizes

Generally want continuum limit, so multiple lattice spacings



Particles need to fit in box, otherwise finite volume errors. Rule of thumb: $m_\pi L > 4$

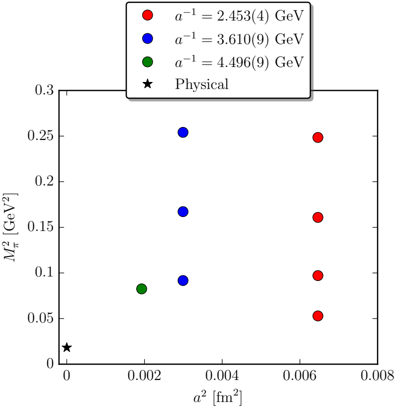


Quarks on the lattice

For the quarks, in most cases, get propagators from columns of inverse Dirac matrix, solve $M\mathbf{x} = \eta$.

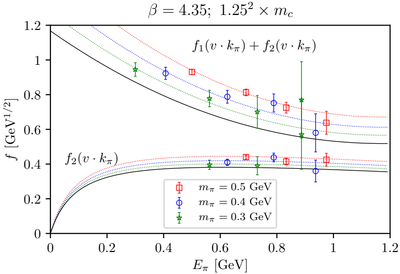
Two issues:

- ★ as am gets small, cost goes up
- ★ as am gets large, discretisation errors increase

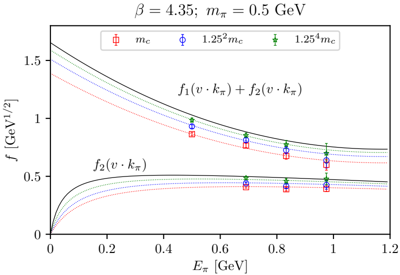


One solution: extrapolate from multiple heavier/lighter masses.

Quark masses create problems



Extrapolation in pion mass from 500 MeV to physical value 135 MeV



Extrapolation in $1/m_b$ to physical mass ~ 4.2 GeV

Heavy quarks

Alternatively, for heavy quarks use effective actions. We consider relativistic heavy quarks (RHQ) and Nonrelativistic QCD (NRQCD). In NRQCD:

$$G_b(\mathbf{x}, t+1) = e^{-aH} G_b(\mathbf{x}, t)$$

with

$$\begin{aligned}
aH &= aH_0 + a\delta H \\
aH_0 &= -\frac{\Delta^{(2)}}{2am_b} \\
a\delta H &= -c_1 \frac{\left(\Delta^{(2)}\right)^2}{8\left(am_b\right)^3} + c_2 \frac{i}{8\left(am_b\right)^2}\left(\tilde{\nabla}\cdot\tilde{\mathbf{E}} - \tilde{\mathbf{E}}\cdot\tilde{\nabla}\right) \\
&\quad -c_3 \frac{1}{8\left(am_b\right)^3}\sigma\cdot\left(\tilde{\nabla}\times\tilde{\mathbf{E}} - \tilde{\mathbf{E}}\times\tilde{\nabla}\right) \\
&\quad -\ c_4\ \frac{1}{2am_b}\sigma\cdot\tilde{\mathbf{B}} +\ c_5\ \frac{\Delta^{(4)}}{24am_b} -\ c_6\ \frac{\left(\Delta^{(2)}\right)^2}{16n(am_b)^2}.
\end{aligned}$$

Tetraquarks on the Lattice

Tetraquarks

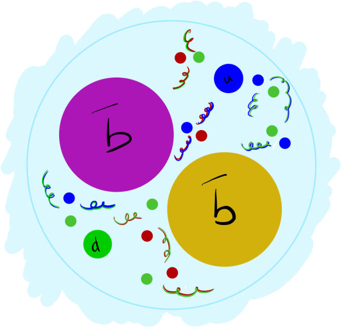
This work w/ Jamie Hudspith, Anthony Francis, Randy Lewis, Kim Maltman

We are interested in states with "good light diquarks". Depending on the anti-diquark content and its configuration, we have access to $J^P = 1^+$ or $J^P = 0^+$ states.

Expectations:

- ★ deeper binding with lighter light diquarks
- ★ deeper binding with heavier heavy diquarks

But there are many states to explore and contradictory claims from models. Predictions of binding and ruling out states both useful for experimentalists.



Fitting our tetraquarks

Construct correlators, $C_{\mathcal{O}_1\mathcal{O}_2}(t) = \sum_n \frac{\langle 0|\mathcal{O}_1|n\rangle\langle n|\mathcal{O}_2|0\rangle}{2E_n} e^{-E_n t}$ from:

$$
\begin{gathered}
D(\Gamma_1,\Gamma_2) = (\psi_a^T C\Gamma_1\phi_b)(\bar{\theta}_a C\Gamma_2\bar{\omega}_b^T), \\
E(\Gamma_1,\Gamma_2) = (\psi_a^T C\Gamma_1\phi_b)(\bar{\theta}_a C\Gamma_2\bar{\omega}_b^T - \bar{\theta}_b C\Gamma_2\bar{\omega}_a^T), \\
M(\Gamma_1,\Gamma_2) = (\bar{\theta}\Gamma_1\psi)(\bar{\omega}\Gamma_2\phi), \qquad N(\Gamma_1,\Gamma_2) = (\bar{\theta}\Gamma_1\phi)(\bar{\omega}\Gamma_2\psi), \\
O(\Gamma_1,\Gamma_2) = (\bar{\omega}\Gamma_1\psi)(\bar{\theta}\Gamma_2\phi), \qquad P(\Gamma_1,\Gamma_2) = (\bar{\omega}\Gamma_1\phi)(\bar{\theta}\Gamma_2\psi).
\end{gathered}
$$

We want to solve a GEVP to get energy levels:

$$C_i(t) = \sum_{j,k} V_{ij}(\tau)^\dagger C_{jk}(t) V_{ki}(\tau)$$

where V is made from columns of the eigenvector solution to:

$$C_{ij}(t)v_j(t) = \lambda_i C_{ij}(t+t_0)v_j(t) \ .$$

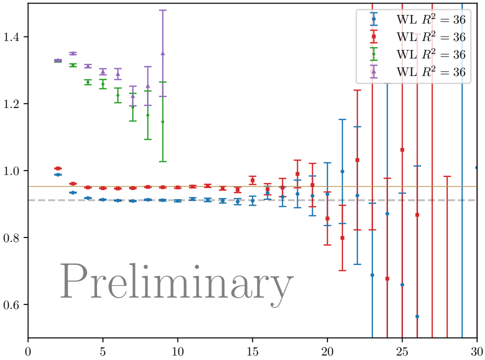
Fitting our tetraquarks

Getting

$$C_i(t) = \sum_{j,k} V_{ij}(\tau)^\dagger C_{jk}(t) V_{ki}(\tau)$$

by solving:

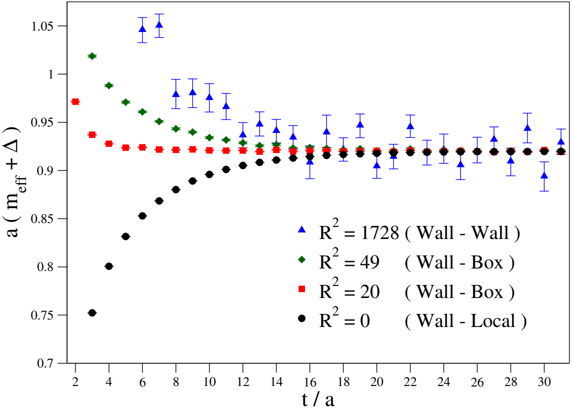
$$C_{ij}(t) v_j(t) = \lambda_i C_{ij}(t + t_0) v_j(t) \ .$$



Recent updates

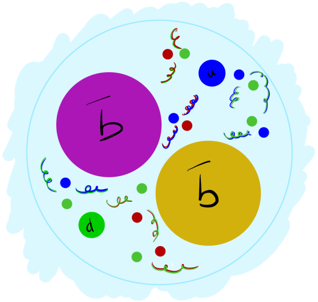
Recent improvement: box-sinks for better overlap with states

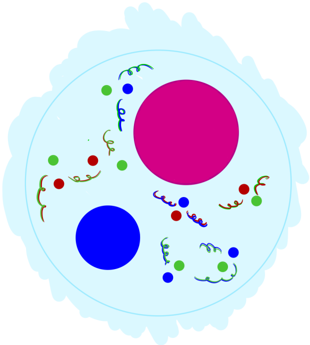
$$S^B(x,t) = \frac{1}{N}\sum_{r^2 \le R^2} S(x+r,t)$$



QCD

One final complication: NRQCD explicitly removes mass from Lagrangian, so all energies are shifted, but not energy differences, so cannot just measure tetraquark mass.



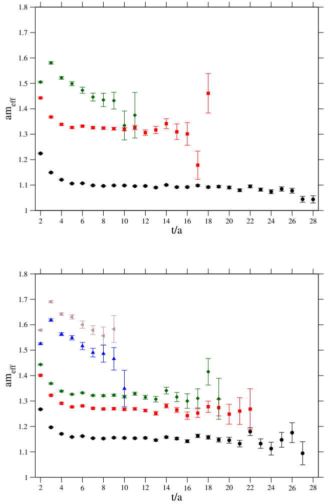


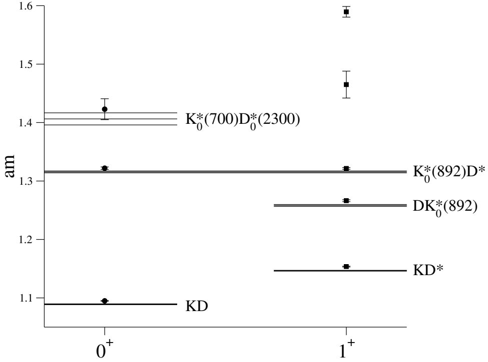
Solution: fit meson correlators, determine energy difference between tetraquark and thresholds.

Doubly-heavy tetraquarks

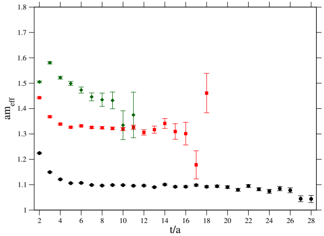
- ★ Phenomenology → bound doubly-bottom tetraquarks
  - ▶ In heavy Q limit, anti-diquark behaves like single heavy quark
    - ○ Evident from small meson & baryon hyperfine splittings
  - ▶ good-light-diquark → $J^P = 1^+$ tetraquark most favourable
    - ○ i.e. should expect deepest binding
- ★ Recent years has seen progress in lattice QCD calculations of tetraquarks with $J^P = 1^+$
  - ▶ Static $\bar{b}\bar{b}$ potentials:
    - ○ P. Bicudo & M. Wagner [1209.6274]
    - ○ Z. S. Brown & K. Orginos [1210.1953]
    - ○ P. Bicudo, J. Scheunert & M. Wagner [1612.02758]
  - ▶ NRQCD $\bar{b}\bar{b}$:
    - ○ A. Francis, R. J. Hudspith, R. Lewis, K. Maltman [1607.05214]
    - ○ P. Junnarkar, N. Mathur & M. Padmanath [1810.12285]
    - ○ L. Leskovec, S. Meinel, M. Pflaumer & M. Wagner [1904.04197]
  - ▶ RHQ & NRQCD $\bar{c}\bar{b}$,$\bar{s}\bar{b}$,$s\bar{c}$:
    - ○ R. J. Hudspith, BC, A. Francis, R. Lewis, K. Maltman [2006.14294]

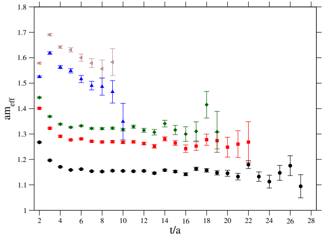
$ud\bar{s}\bar{c}$ tetraquarks

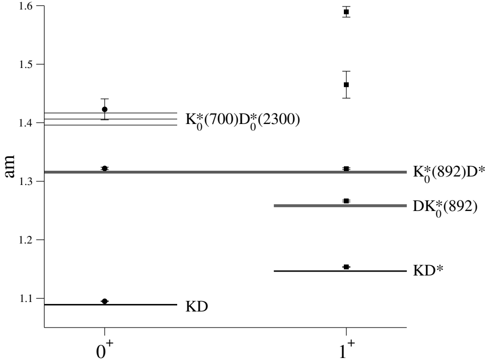




$ud\bar{s}\bar{c}$ tetraquarks

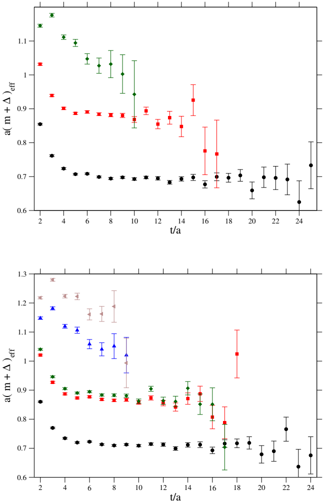


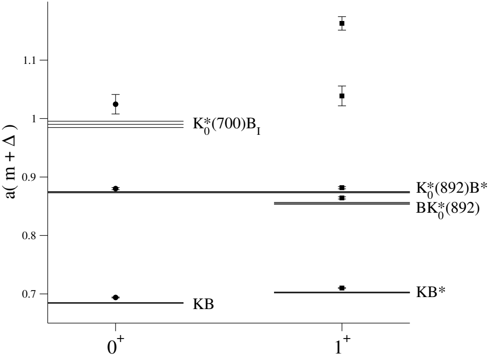




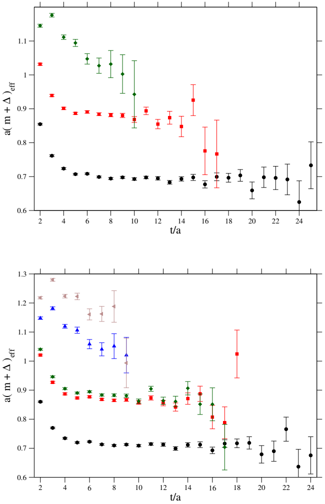
- ★ No evidence of tetraquarks in 0^+ or 1^+ channels

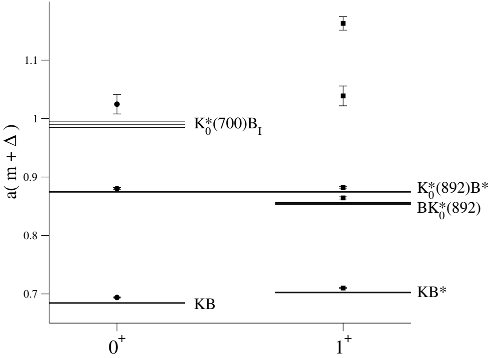
$ud\bar{s}\bar{b}$ tetraquarks





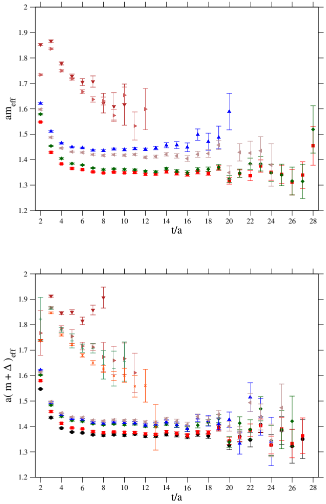
$ud\bar{s}\bar{b}$ tetraquarks

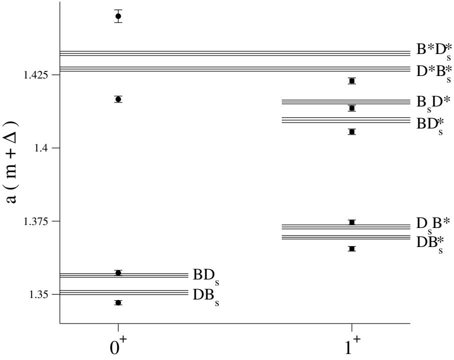




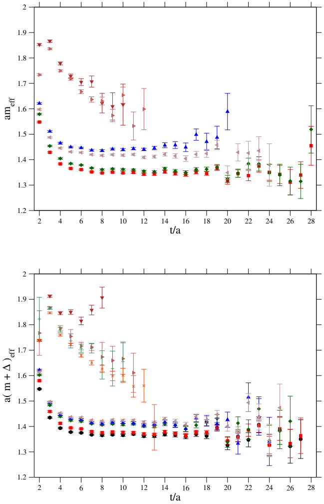
- ★ No evidence of tetraquarks in 0^+ or 1^+ channels

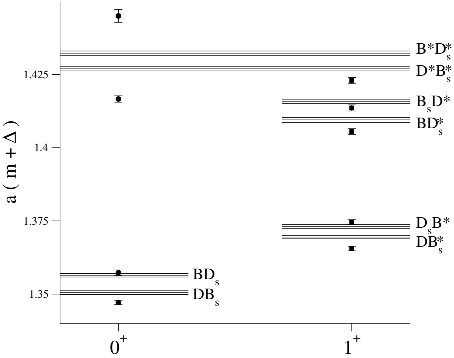
$\ell s\bar{c}\bar{b}$ tetraquarks





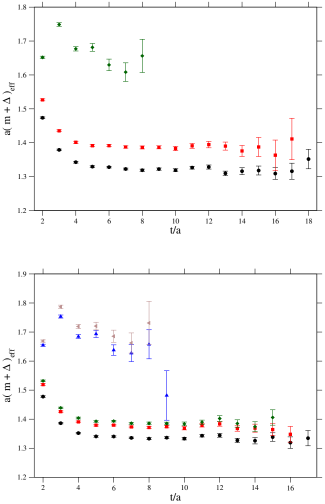
$\ell s\bar{c}\bar{b}$ tetraquarks

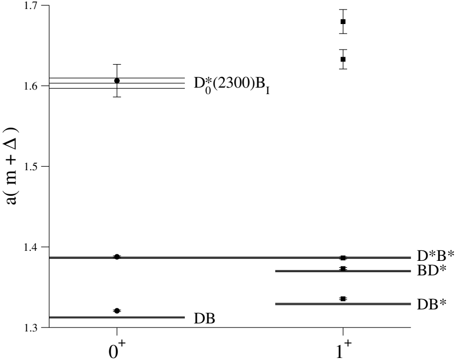




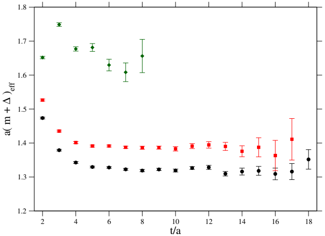
★ No evidence of tetraquarks in 0^+ or 1^+ channels

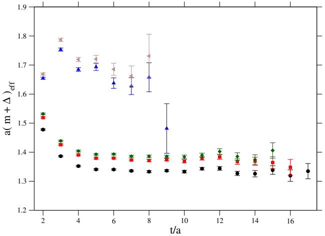
$ud\bar{c}\bar{b}$

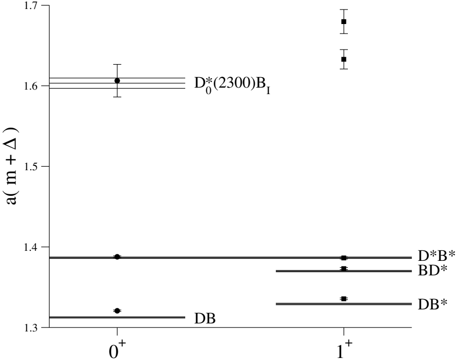

a(m + Δ)$_{eff}$
t/a
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2
4
6
8
10
12
14
16
18


a(m + Δ)
1.3
1.4
1.5
1.6
1.7
$D_0^*(2300)B_I$
D*B*
BD*
DB*
DB
0^+
1^+

$ud\bar{c}\bar{b}$

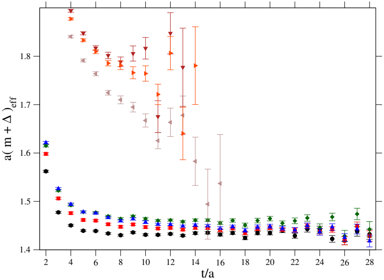


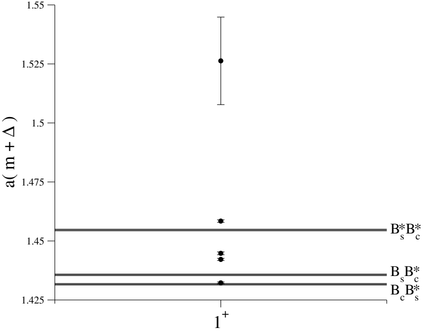




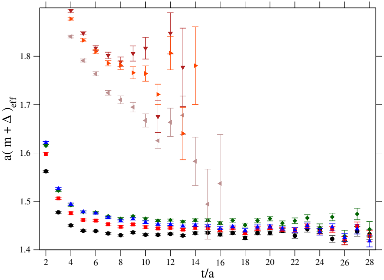
★ No evidence of tetraquarks in 0^+ or 1^+ channels

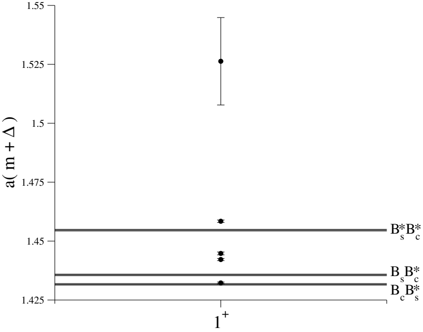
$sc\bar{b}\bar{b}$


a(m + Δ)eff
1.8
1.7
1.6
1.5
1.4
2
4
6
8
10
12
14
16
18
20
22
24
26
28
t/a


a(m + Δ)
1.55
1.525
1.5
1.475
1.45
1.425
B*sB*c
BsB*c
BcB*s
1+

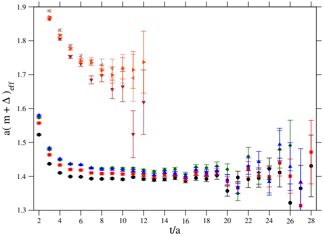
$sc\bar{b}\bar{b}$



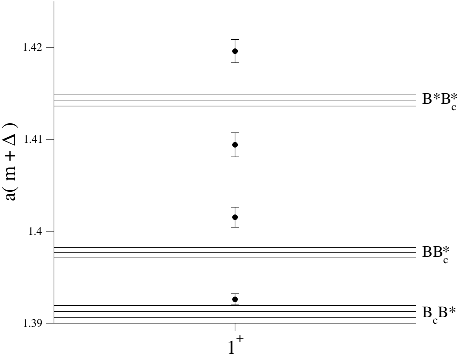


★ No evidence of tetraquarks in 0^+ or 1^+ channels

$uc\bar{b}\bar{b}$

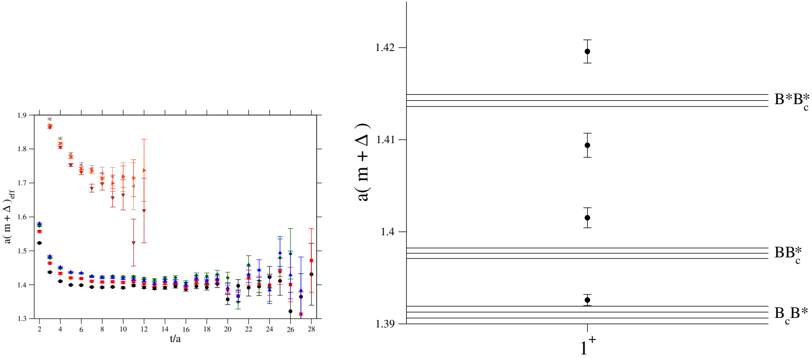


$a(m+\Delta)_{eff}$
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2
4
6
8
10
12
14
16
18
20
22
24
26
28
t/a



$a(m+\Delta)$
1.39
1.4
1.41
1.42
$B^*B^*_c$
BB^*_c
B_cB^*
1^+

$uc\bar{b}\bar{b}$

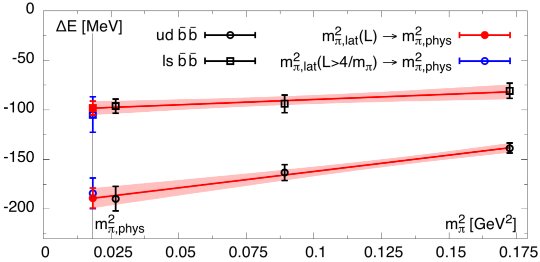


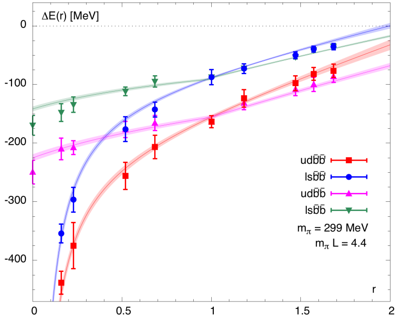
★ No evidence of tetraquarks in 0^+ or 1^+ channels

Doubly-bottom tetraquarks

Francis *et al.* [1607.05214]

- ★ $ud\bar{b}\bar{b}$ clearly bound
- ★ Multiple lattice groups also find evidence of binding

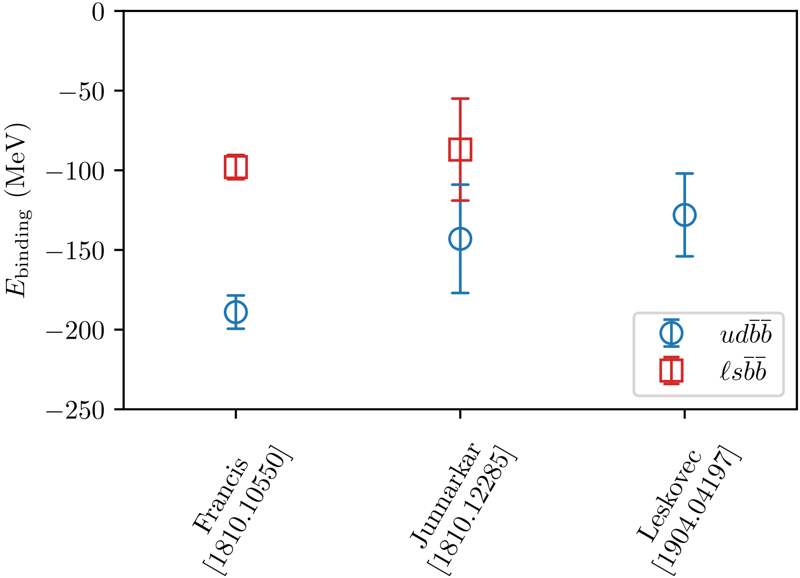




- ★ Binding increases with increasing heavy quark mass

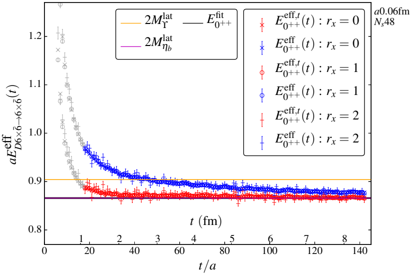
Francis *et al.* [1810.10550]

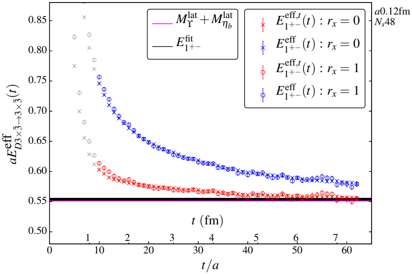
Binding energy comparisons

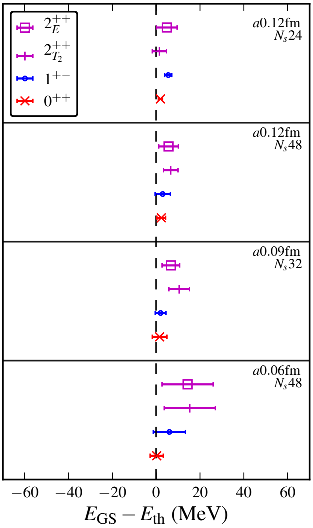


$bb\bar{b}\bar{b}$ tetraquarks

C. Hughes, E. Eichten and C. T. H. Davies [1710.03236]





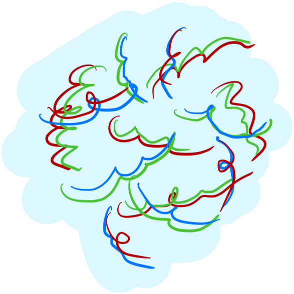


Other Exotics on the Lattice

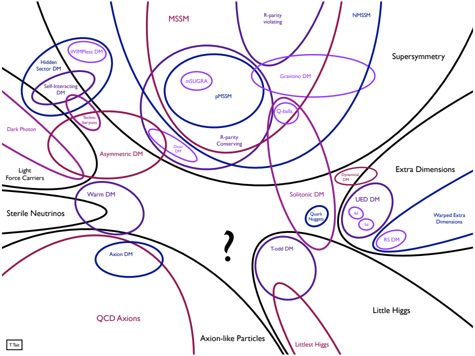
Glueballs

In QCD, the requirement for colourless objects does not rule out bound states of gluons called glueballs. Despite no strong experimental evidence, they can be studied on the lattice.

- ★ Multiple lattice calculations in *quenched approximation* find glueball masses:
  - ▶ $J^{PC} = 0^{++} \quad \sim 1500 - 1800 \text{ MeV}$
  - ▶ $J^{PC} = 2^{++} \quad \sim 2250 - 2400 \text{ MeV}$
    - ○ [hep-lat/9304012], [hep-lat/9704011], [hep-lat/0510074]
- ★ In mass range of some exotic mesons
  - ▶ Could also be tetraquarks!
- ★ Unquenching might have little effect
  - ▶ [1005.2473], [1208.1858]



Dark matter



T. Tait [1310.8642]

- ★ Dark matter: more abundant than baryonic matter
- ★ Properties are unknown
- ★ Possibility: dark matter is strongly-coupled composite?
- ★ Since we lack knowledge of DM properties, we can explore different models
  - ▶ (Unless that model is phenomenologically ruled out)

One example is SU(2) w/ single flavour of "quark", Francis et al., [1809.09117]

Dark matter

This work w/ Anthony Francis, Randy Lewis, Enrico Rinaldi, Sean Tulin

Our proposal: with the $SU(2)$, $N_f = 1$ model, calculate scattering rates. Two methods:

Luscher's Method

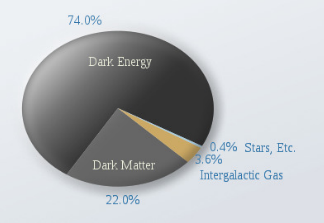
- ★ Calculate the energy from *temporal* correlators
- ★ Energy relates to scattering phase shift

HALQCD Method

- ★ Wave function from *spatial correlator*
- ★ Phase shifts extracted from wavefunction

Different approaches but same scattering physics, so sensible cross-check.

Dark matter



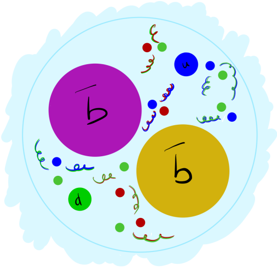
Aflafla1, CC0

Existing lattice models:

- ★ $SU(2)$ $N_f > 1$
  - ▸ very different physics
- ★ $SU(3)$
- ★ $SU(4)$ w/ $N_f = 4$, stealth dark matter
- ★ $SO(4)$

Summary

- ★ $ud\bar{b}\bar{b}$ state studied by various groups: agreement bound $\mathcal{O}(100)$ MeV
- ★ Experimental search worthwhile for $ud\bar{b}\bar{b}$
- ★ Evidence also: $\ell s\bar{b}\bar{b}$
- ★ None of the other states we have explored have evidence of strongly-bound tetraquarks.
- ★ Models claiming deep binding in these channels are ruled out



- ★ The lattice useful for other exotics: glueballs, hybrids
- ★ Lattice interest in strongly-coupled dark matter theories for info on spectrum, scattering.

Thank you!